GCSE Maths – Algebra

Solving Linear Inequalities

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions involving linear inequalities. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Solve the inequality $3x - 4 < 18 + x$. Present your answer in a number line.

Step 1: Rearrange the inequality to move all terms containing the unknown to one side of the equation.

$$3x - 4 < 18 + x$$

Subtract x from each side of the equation:

$$3x - 4 - x < 18 + x - x$$
$$2x - 4 < 18$$

Step 2: Solve for x. If you multiply or divide by -1 remember that the sign of the inequality flips direction.

$$2x - 4 < 18$$

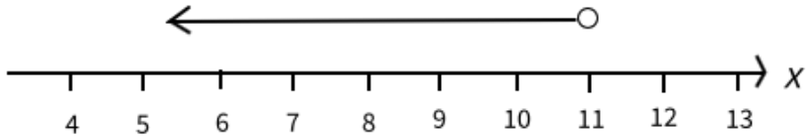
Add 4 to both sides of the equation to eliminate the constant term from the left-hand side:

$$2x < 22$$

Divide both sides of the equation by 2 to remove the coefficient of x:

$$x < 11$$

Step 3: Draw a number line to illustrate the answer. If the sign used is $\leq$ or $\geq$, a solid circle should be used. Otherwise, an open circle should be used.



Guided Example

Solve the inequality $5 \geq 13 - 2x$. Present your answer in a number line.

Step 1: Ensure only the unknown is present on one side of the inequality.

$5 \geqslant 13 - 2x$ (+2x, +2x)
$2x + 5 \geqslant 13$ (−5, −5)
$2x \geqslant 8$

Step 2: Solve for x.

$2x \geqslant 8$
$x \geqslant 4$

Step 3: Draw a number line to illustrate the answer.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Solve the following inequalities and present your answer in a number line:

a) $2x + 1 \geq 5 + x$

$-x$

$x + 1 \geq 5$

-1

$x \geq 4$

b) $2(x + 2) < -14 - x$

$2x + 4 < -14 - x$

$+x$

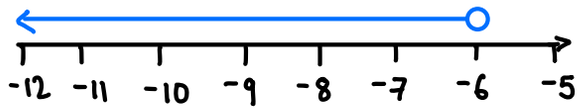
$3x + 4 < -14$

-4

$3x < -18$

$\div 3$

$x < -6$



c) $x - 6 \geq 4x + 3$

$-x$

$-6 \geq 3x + 3$

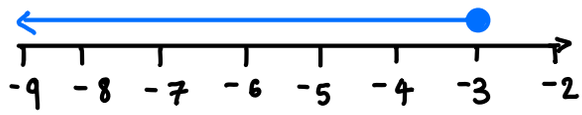
-3

$-9 \geq 3x$

$\div 3$

$-3 \geq x$

$x \leq -3$



d) $-4(x - 5) \leq -3(2x - 7)$

$-4x + 20 \leq -6x + 21$

$+6x$

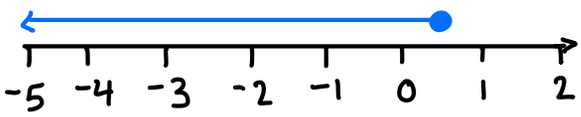
$2x + 20 \leq 21$

-20

$2x \leq 1$

$\div 2$

$x \leq 1/2$



Section B

Worked Example

Solve the inequality $29 < 5 - 3x \leq 35$.

Present your answer in a number line and list the integer solutions.

Step 1: Split the inequality into two separate inequality parts.

a) $29 < 5 - 3x$

b) $5 - 3x \leq 35$

Step 2: Solve each inequality separately. Put the answers together to find the values which x can take.

a) $29 < 5 - 3x$

Add $3x$ to both sides of the equation: $3x + 29 < 5$
Subtract 29 from both sides of the equation: $3x < -24$
Divide 3 from both sides of the equation: $\boldsymbol{x < -8}$

b) $5 - 3x \leq 35$

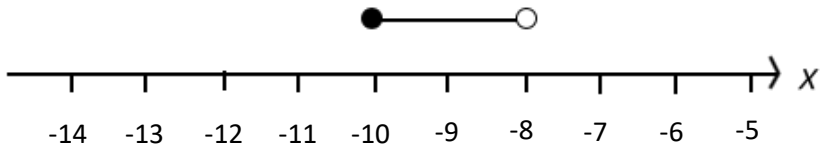
Subtract 5 from both sides of the equation: $-3x \leq 30$
Divide both sides of the equation by -3 (Note: Dividing the inequality by a negative number will change the direction of the inequality symbol):

$$\boldsymbol{x \geq -10}$$

Putting the inequalities together: $\boldsymbol{-10 \leq x < -8}$

Step 3: Draw a number line to illustrate the inequality region.

The number line has a filled circle at -10 since x can take value -10. There is a non-filled circle at -8 since x cannot take the value -8.



Step 4: List the set of integers which satisfy the number line.

The integers which satisfy $\boldsymbol{-10 \leq x < -8}$ are $\boldsymbol{x = -10}$ and $\boldsymbol{x = -9}$.

Guided Example

Solve the inequality $-3 \leq 2x - 1 \leq 5$.

Present your answer in a number line and list down the integer solutions.

Step 1: Split the inequality into two separate inequality parts.

a) $-3 \leq 2x - 1$

b) $2x - 1 \leq 5$

Step 2: Solve each inequality separately. Put the answers together to find the values which x can take.

a) $-3 \leq 2x - 1$

$-2 \leq 2x$

$-1 \leq x$

b) $2x - 1 \leq 5$

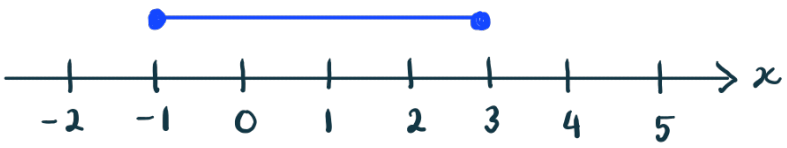
$2x \leq 6$

$x \leq 3$

Putting the inequalities together:

$-1 \leq x \leq 3$

Step 3: Draw a number line to illustrate the inequality region.



Step 4: List the set of integers which satisfy the number line.

The integers which satisfy $-1 \leq x \leq 3$ are $-1, 0, 1, 2$ and 3.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

2. Solve the following inequalities. List the integers in each solution set.

a) $1 \leq 2y - 1 \leq 5$

Step 1: (i) $1 \leq 2y - 1$

(ii) $2y - 1 \leq 5$

Step 2: (i) $2 \leq 2y$ (ii) $2y \leq 6$

$1 \leq y$ $y \leq 3$

$1 \leq y \leq 3$

Step 3:

Step 4: The integers which satisfy $1 \leq y \leq 3$ are 1, 2 and 3.

a) $-6 < p + 6 \leq 8$

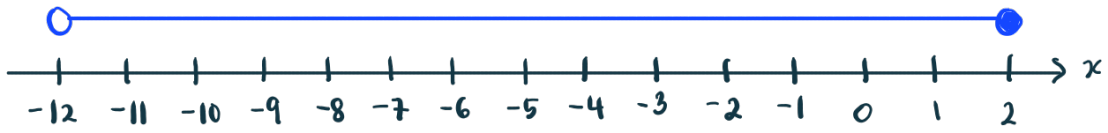
Step 1: (i) $-6 < p + 6$

(ii) $p + 6 \leq 8$

Step 2: (i) $-12 < p$ (ii) $p \leq 2$

$-12 < p \leq 2$

Step 3:



Step 4: The integers which satisfy $-12 < p \leq 2$ are $-11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1$ and 2.

Section C – Higher only

Worked Example

Solve the inequality $2x < 5 + 4x - y$.

Present your answer on a graph.

Step 1: Rearrange the inequality to ensure only y is present on the left-hand side and x is on the right-hand side. The sign for y should be positive and its coefficient should be 1.

$$2x < 5 + 4x - y$$

Add y to both sides of the inequality: $y + 2x < 5 + 4x$
Subtract $2x$ from both sides of the inequality: $y < 5 + 2x$

Step 2: Replace the inequality sign by '=' to help plot the required graph. Find the x –intercept and the y –intercept to find coordinates on the line.

Here, we are interested in the graph $y = 5 + 2x$.

- *To find the* x *–intercept, set* $y = 0$*:*

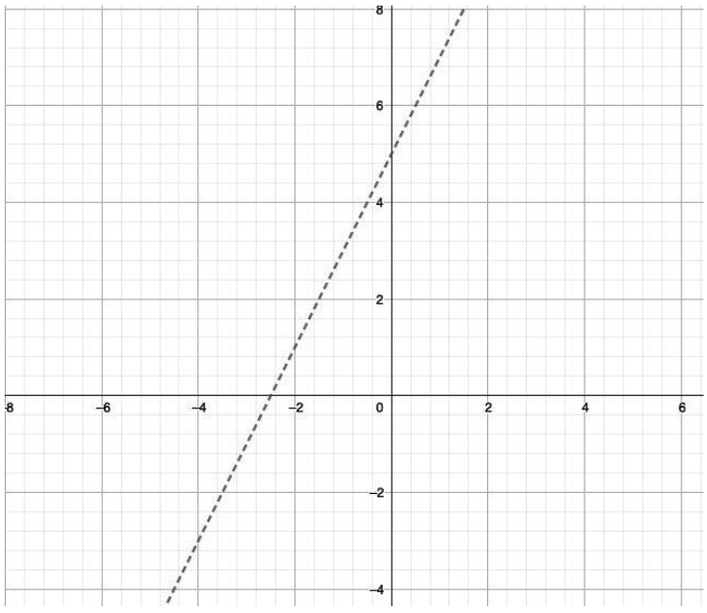
$$y = 5 + 2x$$
$$0 = 5 + 2x$$
$$x = -2.5$$

- *To find the* y *–intercept, set* $x = 0$*:*

$$y = 5 + 2\,(0)$$
$$y = 5$$

Step 3: Plot both the x –intercept and the y –intercept and draw the line equivalent to the inequality. If $<$ or $>$ sign is used, the line drawn should be a dotted line. However, if the sign used is $\leq$ or $\geq$, a solid line should be drawn.

From above, we can plot the points $(-2.5,0)$ *and* $(0,5)$.



Step 4: Shade the region which satisfies the inequality.

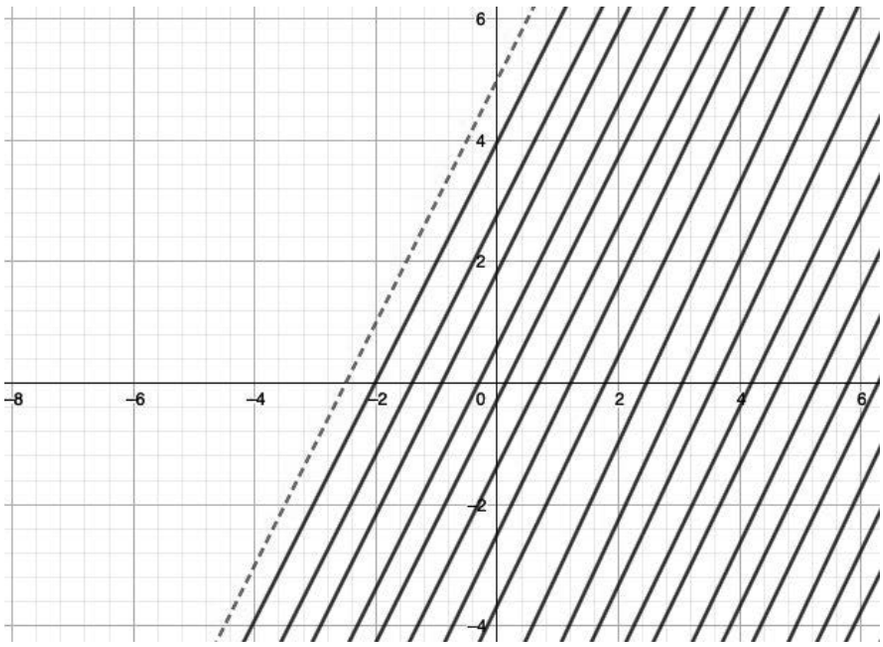
To help identify the correct region, choose a coordinate and substitute it into the inequality. If the coordinate satisfies the inequality, the region where the coordinate lies should be shaded.

For $\boldsymbol{y < 5 + 2x}$, the value of y should always be lower than the dotted line. Since y is lower in the lower region of the graph, that region should be shaded.

Alternatively, we can also choose a point in the lower region to check our answer. For instance, if we choose $(2, -2)$ and substitute it in the inequality (as shown below), we will get a correct statement.

$$y < 5 + 2x$$
$$-2 < 5 + 2(2)$$
$$-2 < 9$$

This means that the section containing $(2, -2)$ should be shaded.



Guided Example

Solve the inequality $-y - 4 \geq 3(x + 1)$.

Present your answer in a graph.

Step 1: Rearrange the inequality to ensure only y is present on the left-hand side and x is on the right-hand side. The sign for y should be positive and its coefficient should be 1.

$-y - 4 \geq 3x + 3$

$+y$

$-4 \geq 3x + 3 + y$

$-3x - 3$

$-3x - 7 \geq y$

$y \leq -3x - 7$

Step 2: Replace the inequality sign by '=' to help plot the required graph. Find the x –intercept and the y –intercept to find coordinates on the line.

$y = -3x - 7$

When $x = 0$:

$y = -3(0) - 7$

$y = -7$

$(0, -7)$

When $y = 0$

$0 = -3x - 7$

$+3x$

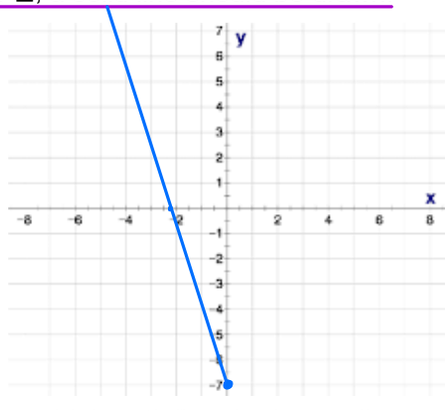
$3x = -7$

$\div 3$

$x = -7/3$

$(-7/3, 0)$

Step 3: Plot both the x –intercept and the y –intercept and draw the line equivalent to the inequality. If $<$ or $>$ sign is used, the line drawn should be a dotted line. However, if the sign used is $\leq$ or $\geq$, a solid line should be drawn.

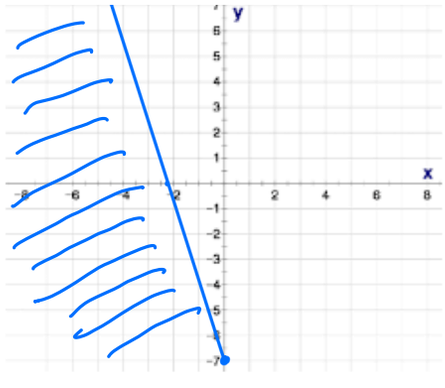


Step 4: Shade the region which satisfies the inequality.
To help identify the correct region, choose a coordinate and substitute it into the inequality. If the coordinate satisfies the inequality, the region where the coordinate lies should be shaded.

choose $(0,0)$ $-3(0) - 7 = -7$ $0 \not\geq -7$

$y = 0$

$(0,0)$ should not be shaded as $y \geq -3x - 7$ not $y \leq -3x - 7$



Now it's your turn!
If you get stuck, look back at the worked and guided examples.

3. Solve the following inequalities and present your answers in a graph.

a) $3x - y < 8y + 2$

-2

$3x - 2 - y < 8y$

$+y$

$3x - 2 < 9y$

$\div 9$

$\frac{1}{3}x - \frac{2}{9} < y$

$y > \frac{1}{3}x - \frac{2}{9}$

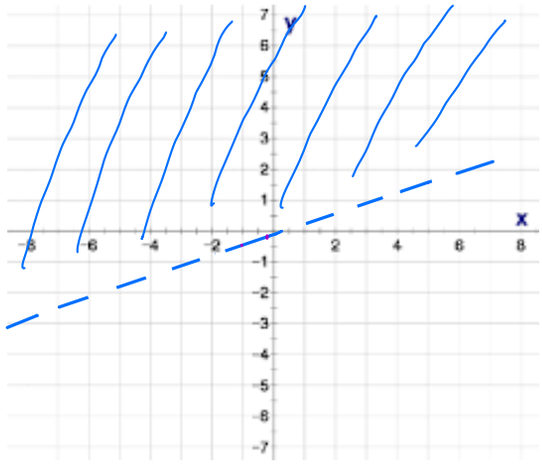
When $x = 0$,
$y = \frac{1}{3}(0) - \frac{2}{9}$
$y = -\frac{2}{9}$
$(0, -2/9)$

When $y = 0$
$0 = \frac{1}{3}x - \frac{2}{9}$
$+\frac{2}{9}$
$\frac{2}{9} = \frac{1}{3}x$
$\times 3$
$\frac{2}{3} = x$
$(2/3, 0)$

As it is $<$, draw a dotted line.

$(0,0)$ $\frac{1}{3}(0) > -\frac{2}{9}$

$0 > -\frac{2}{9}$, so $(0,0)$ is shaded



b) $2g + 2m \geq 7g - 10$

Treat m as y and g as x

$-2g$

$2m \geq 5g - 10$

$\div 2$

$m \geq \frac{5}{2}g - 5$

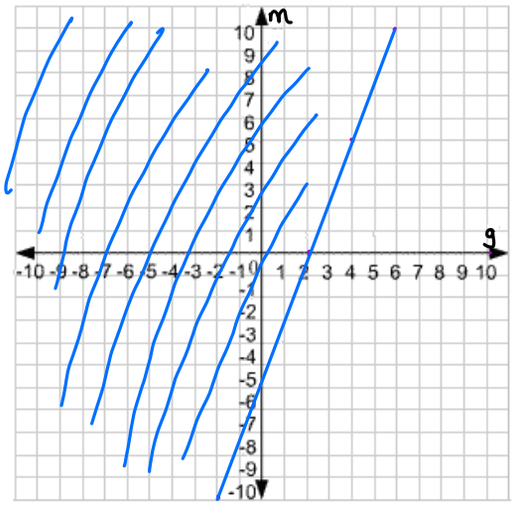
When $g = 0$,
$m = \frac{5}{2}(0) - 5$
$m = -5$
$(0, -5)$

When $m = 0$,
$0 = \frac{5}{2}g - 5$
$\frac{5}{2}g = 5$
$\div \frac{5}{2}$
$g = 2$
$(2, 0)$

As it is $\geq$, solid line.

$(0,0)$: $\frac{5}{2} \times 0 - 5 = -5$

$0 > -5$ so $(0,0)$ is shaded in



c) $2a + 4b \geq 3a - 4$ Treat a as y and b as x

$-2a$

$4b \geqslant a - 4$

$+4$

$4b + 4 \geqslant a$

$a \leqslant 4b + 4$

As $\leqslant$, solid line

When $b = 0$

$a = 4(0) + 4$

$a = 4$

$(0, 4)$

When $a = 0$

$0 = 4b + 4$

$4b = -4$

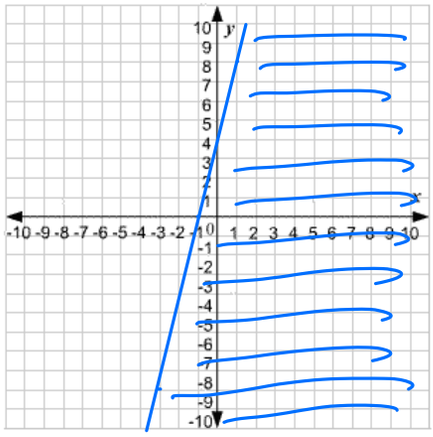
$b = -1$

$(-1, 0)$

$(0,0)$

$4(0) + 4 = 4$

$0 < 4$ so $(0,0)$ is shaded in



d) $-4x - 3y < -12$

$+3y$

$-4x < -12 + 3y$

$+12$

$-4x + 12 < 3y$

$\div 3$

$\frac{-4}{3}x + 4 < y$

$y > \frac{-4}{3}x + 4$

As $>$, dotted line.

$(0,0)$

$\frac{-4}{3}(0) + 4 = 4$

$0 < 4$ so $(0,0)$ is not shaded in

When $x = 0$

$y = \frac{-4}{3}(0) + 4$

$y = 4$

$(0, 4)$

When $y = 0$

$0 = \frac{-4}{3}x + 4$

$\frac{4}{3}x = 4$

$x = 3$

$(3, 0)$